# Origin of Multifractality in Solar Wind Turbulence: the Role of Current Sheets Supplementary Material

November 22, 2022



## 1 Introduction

In order to confirm our results, in addition to the two time series analyzed in the paper, we show here the results for another pair of time series of solar wind magnetic field. We apply the same methodology described in the paper for the data of 2009 February 24 and 2003 February 1. The time series of February 24 is characterized by the presence of large-scale current sheets. Meanwhile, the time series for February 1 is characterized by the absence of current sheets.

Figure 1 shows the two new solar wind magnetic field time series, detected through the Fluxgate Magnetometer (FGM) onboard Cluster-1. The data have a length of 24 hours with a cadence of 22 Hz. Due to the high nonstationarity present in the two time series (the Hurst exponents are $h_{cs} = 1.277$ for February 24 and $h_{ncs} = 1.178$ for February 1), it was necessary to differentiate both of them before applying the MF-DFA method.

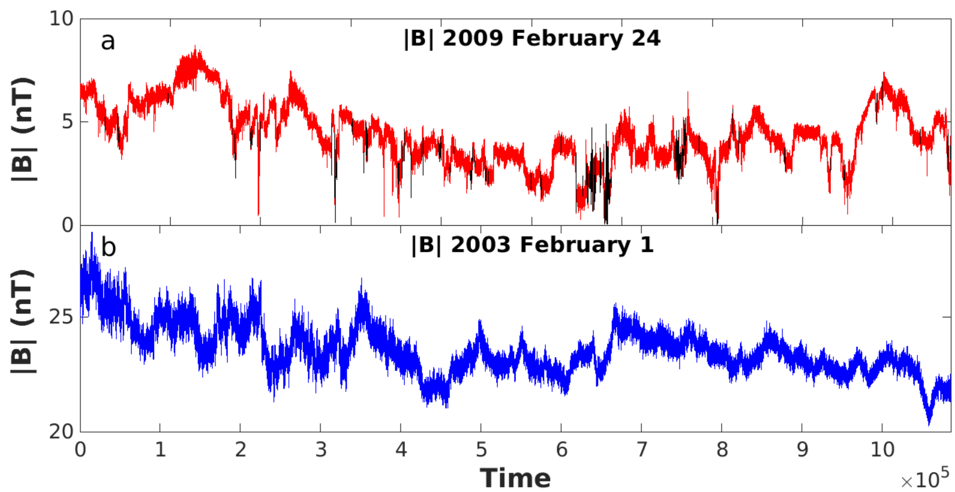


Figure 1: Solar wind time series of $|B|$ (a) for 2009 February 24 (red), containing current sheets (black) and (b) for 2003 February 1 (blue), without current sheets.

## 2 MF-DFA of the $|B|$ time series

Figure 2 shows the multifractal spectra for the time-differenced series of the two data sets shown in Fig. 1. Figure 2 evidence the left asymmetry for the multifractal spectrum of February 24 (with current sheets (**CS**)) (red), which indicates that multifractality is predominantly related to large-scale fluctuations. However, for February 1 (blue), a right asymmetry is observed, i.e., the multifractality is more related to the small-scale fluctuations. Now comparing the $\alpha$ width of both spectra, we can see a considerable difference between them ($\Delta\alpha_{cs} \approx 0.46$ for February 24(**CS**) and $\Delta\alpha_{ncs} \approx 0.35$ for February 1). Thus, the singularity spectrum is wider for the time series with current sheets.

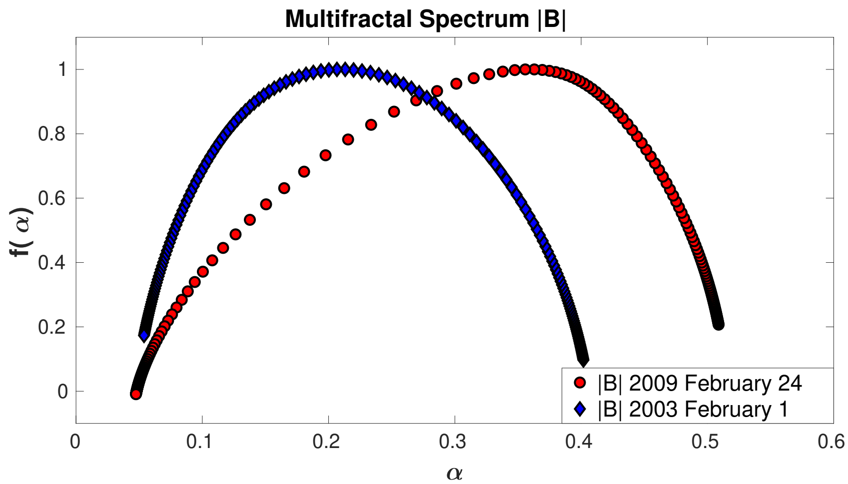


Figure 2: Multifractal spectra of $|B|$ for 2009 February 24 (red) and 2003 February 1 (blue).

# 3 MF-DFA of the Volatility time series

With the goal of enhancing the extreme events present in the series due to current sheets, we applied the volatility to the two data sets shown in Fig. 1. The results are depicted in Fig. 3. Figure 4 shows the corresponding multifractal spectra, with a significant difference of width between them. For February 24(**CS**), $\Delta\alpha_{cs} \approx 0.84$, while for February 1, $\Delta\alpha_{ncs} \approx 0.56$. Thus, the volatility has enhanced the difference between the spectra and highlighted the role of current sheets in multifractality.

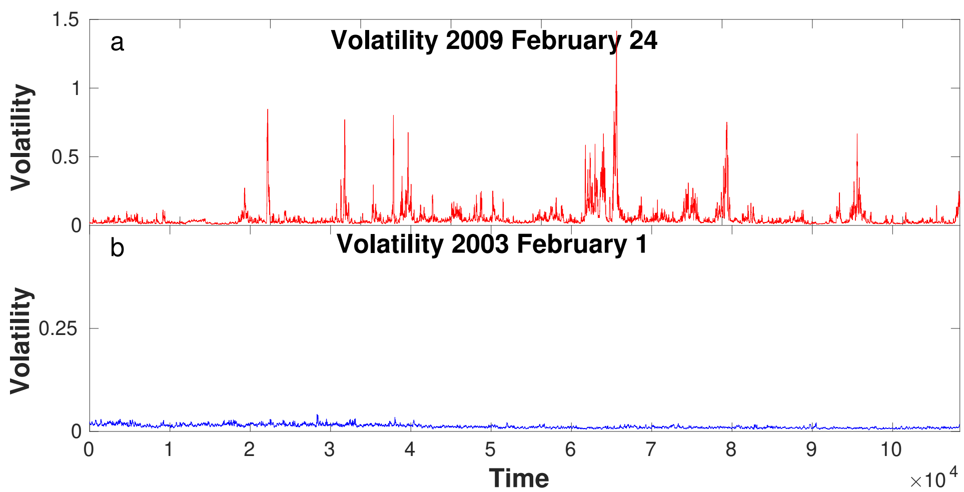


Figure 3: Volatility of solar wind magnetic field time series for (a) 2009 February 24 (red), and (b) 2003 February 1 (blue).

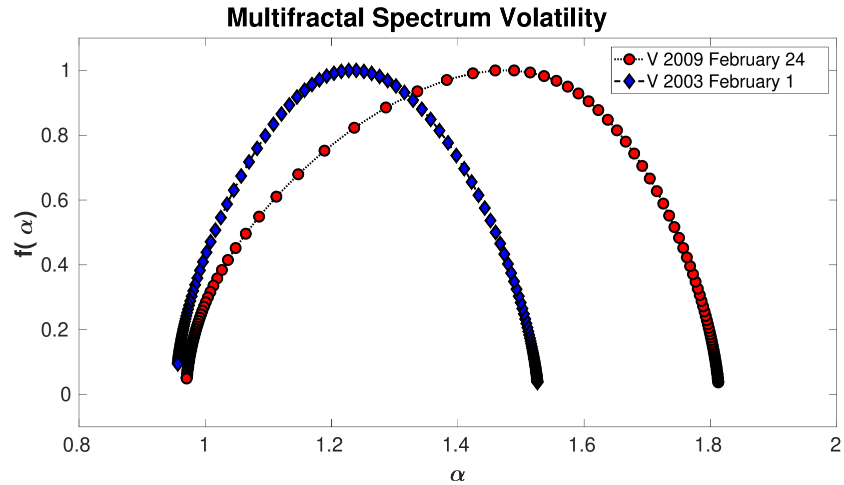


Figure 4: Multifractal spectra for the volatility in 2009 February 24 (red), and 2003 February 1 (blue).

# 4 MF-DFA OF SURROGATE TIME SERIES

In order to identify the origin of the multifractal behavior, we have applied the *surrogate method* for all $|B|$ and volatility time series. After obtaining the *shuffled* and the *random phases* time series for each data set we computed the multifractal spectra.

Figure 5 shows the multifractal spectra for February 24(**CS**) and its surrogates. Both surrogate spectra have a significant width reduction when compared with the original. However, the random phases multifractal spectrum has a stronger reduction. This behavior confirms the idea of a larger contribution of the PDF to this multifractal process, as a consequence of the current sheets.

Now, looking at the multifractal spectra of the solar wind magnetic field without current sheets (Fig. 6), we see a different behavior. Even though there is a width reduction in both surrogates spectra, we observe a larger decrease in the shuffled spectrum. That is, the contribution from long-range correlations to multifractality is more dominant in the time series without current sheets. This result is consistent with what is proposed in the main paper.

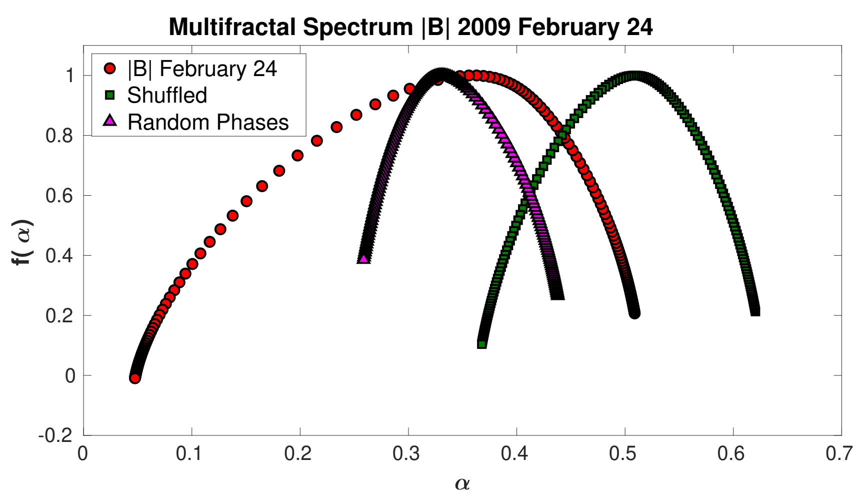


Figure 5: Multifractal spectra of $|B|$ for 2009 February 24 (red) and the respective surrogates: shuffled (green), and random phases (magenta).

As was done with the paper examples, we also applied the surrogate method to the volatility time series. As expected, the same general behavior is observed in the multifractal spectra.

Figure 7 confirms a major influence of extreme events to multifractality for February 24(**CS**), since $\Delta\alpha$ of the random phases surrogate has a smaller value than the one from the shuffled surrogate. Figure 8 exhibits the singularity spectra for the volatility time series of February 1. Unsurprisingly, these spectra also follow the same general behavior as the corresponding spectra obtained from the $|B|$ time series. As expected, the results from Fig.8 highlight the long-range correlations influence on the multifractality of the volatility time series.

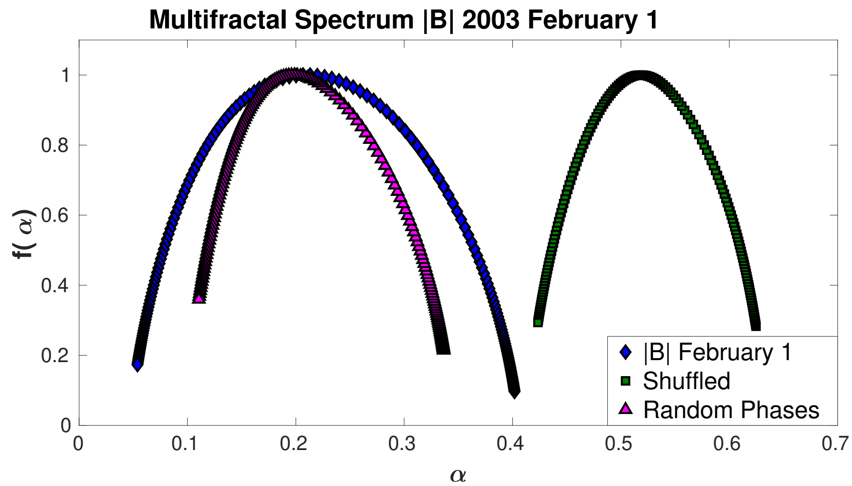


Figure 6: Multifractal spectra of $|B|$ for 2003 February 1 (blue) and the respective surrogates: shuffled (green), and random phases (blue).

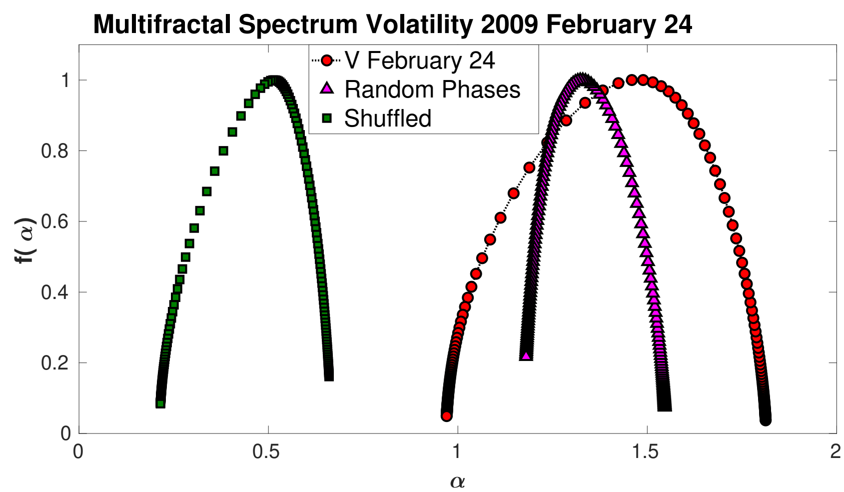


Figure 7: Multifractal spectra for the volatility of 2009 February 24 (red) and the respective surrogates: shuffled (green), and random phases (magenta).

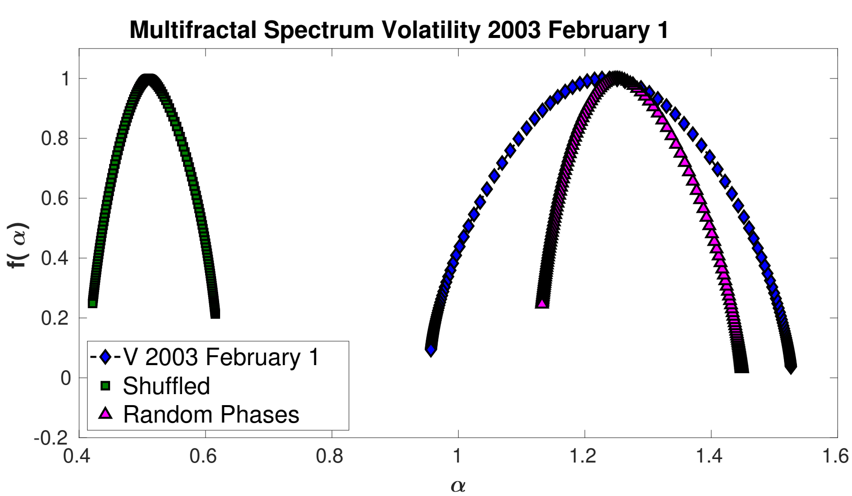


Figure 8: Multifractal spectra for the volatility of 2003 February 1 (blue) and the respective surrogates: shuffled (green), and random phases (magenta).

A summary of all values provided by the multifractal analysis can be found on Tables 1 and 2. Table 1 shows the variation of the $\alpha$ width, $\Delta\alpha$, while Table 2 emphasizes the spectral asymmetry.

Table 1: Width of $\alpha$, $\Delta\alpha$, for magnetic field and volatilities of 2009 February 24 (**CS**) and 2003 February 1.

| | February 24 (**CS**) | | February 1 | |
|---|---|---|---|---|
| | $\|B\|$ | Volatility | $\|B\|$ | Volatility |
| Original | 0.4614 | 0.84202 | 0.3480 | 0.5695 |
| Shuffle | 0.2527 | 0.44506 | 0.20122 | 0.1941 |
| Random Phases | 0.17934 | 0.36611 | 0.2260 | 0.3161 |

Table 2: Spectral asymmetry, $A$, for magnetic field and volatilities of 2009 February 24 (**CS**) and 2003 February 1.

| | February 24 (**CS**) | | February 1 | |
|---|---|---|---|---|
| | $\|B\|$ | Volatility | $\|B\|$ | Volatility |
| Original | 0.4626 | 0.62344 | 1.18403 | 1.0156 |
| Shuffle | 0.7764 | 0.47176 | 1.1017 | 1.1842 |
| Random Phases | 1.4568 | 1.44489 | 1.5630 | 1.6386 |

In summary, in order to reinforce the conclusions of the main work we have applied the *MF-DFA method* on two new solar wind magnetic field time series. The new findings of this supplementary material confirm our theoretical conclusions described in the paper. In other words, the presence of current sheets indeed impacts the turbulent process of the solar wind and is responsible for the increase of multifractality due to PDFs. In the absence of current sheets, multifractality is mainly due to nonlinear correlations.