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Evaluating Ordinal Pattern Features for 2D Colored Noise Classification

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This study explores the potential of permutation entropy and statistical complexity for analyzing time series and image data of varying dimensions and noise types to extract features for computational vision. We projected one-dimensional colored noise of different sizes and one- and two-dimensional $1/f$ noise with different *embedding* dimensions to observe changes in permutation entropy and statistical complexity. The results of this study provide insights into the usefulness of the permutation entropy and statistical complexity in the analysis of complex time series data for future parameter extraction.

1 Introduction

Ordinal Patterns are a symbolization technique proposed by Brandt and Pompe in 2002 [1]. This technique has been applied in several areas as physics, medicine, biomedicine, and economy [3, 6, 12–14]. In recent years; it has been applied to time series and generalized to two dimensions, allowing the description of entropic complexity representations in a two-dimensional space [7]. The technique can describe complex transitions between different regimes in time series, including coloured noise time $(1/f^\beta)$, where β is the power law spectral index. Generalizations for the 2D approach have been tested in some applications to characterize image data. In this realm, generating 2D coloured noise with a conservation structure is still a challenge [5, 8]. This paper uses Ordinal Patterns to analyze the behaviour of $1/f^\beta$ noise in one and two dimensions (1D and 2D).

1.1 Ordinal Patterns and Associated Features

Ordinal Patterns are a powerful representation for analyzing data across various fields of science. This technique is grounded in Information Theory and involves comparing the evolution of a time series $\boldsymbol{x}$ at fixed intervals τ within an *embedding* dimension n. By identifying patterns within the series and tracking the occurrences of each pattern in a vector, it is possible to measure the complexity of the system's information through the Shannon entropy:

$$H(\boldsymbol{\pi}) = -\sum_{\ell=1}^{n!} p(\pi_\ell) \log p(\pi_\ell), \tag{1}$$

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where $\boldsymbol{\pi}$ is the series of patterns, π_ℓ is any of the $n!$ possible patterns, $p(\pi_\ell)$ is the proportion of a pattern π_ℓ in the sequence. This measure, usually referred to as "Permutation Entropy (PEn)" indicates the amount of information in a system, with greater unpredictability represented by more uniform occurrences of each state. As such, PEn provides a powerful tool for understanding and interpreting complex data sets. Recent studies have highlighted the utility of PEn in detecting patterns in financial data and analyzing a range of biological and physical systems [14].

For illustrative purposes, consider Figure 1, which shows the application of this technique. We present a uniformly-distributed white noise time series, resulting in maximum PEn (in the limit, all patterns will be present in the same proportion). Conversely, a monotonically increasing time series would result in minimal PEn as only one observed state is presented.

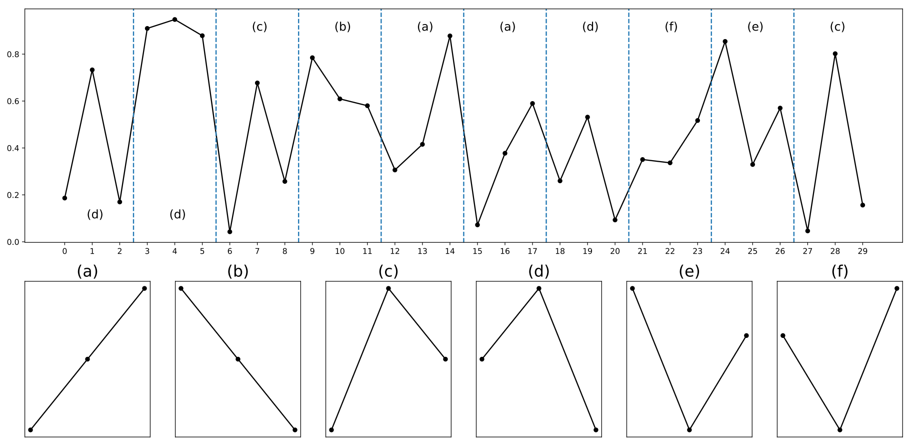

Figure 1: Example of time series permutation. In this example, the window size is $n = 3$ and $\tau = 3$, thus there are 6 permutation patterns (a)-(f). The time series is split into segments of size 3 and every segment is classified. In this example, we have the occurrences of states (a)-(f), respectively: 20 %, 10 %, 20 %, 30 %, 10 %, and 10 %. Therefore, the Permutation Entropy is 1.695.

PEn has proven to be a useful tool for the analysis of time series in several areas of knowledge, allowing the identification of patterns and the prediction of future behavior. Furthermore, PEn can be combined with other data analysis techniques, such as principal component analysis and cluster analysis, to gain a deeper understanding of the complexity of the studied systems. In this sense, the statistical complexity has been proposed as a complementary method [9].

1.2 Colored Noise

Understanding and replicating noise signals is crucial for modeling and simulating physical phenomena, improving signal transmission quality in telecommunications, and various other applications in signal processing and data analysis. Colored noise is a type of noise with a Power Spectral Density (PSD) that is not constant across all frequencies, resulting in a frequency-dependent distribution of energy. The log-log scaling laws between the PSD and the frequency are commonly used to classify noise by its color. The PSD is calculated by taking the modulus of the Fourier Transform coefficients $\widehat{X}_\omega$ of the signal $(x_n)_{0 \le n \le N-1}$, which can be calculated as:

$$\widehat{X}_\omega = \sum_{n=0}^{N-1} x_n e^{-i2\pi\omega n/N}, \quad \omega = 0, 1, 2, \ldots, N-1, \tag{2}$$

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in which N is the number of elements in the series, and ω is the frequency term. The colored noise scaling law relationship is expressed as follows:

$$\ln \|\widehat{X}_\omega\| \approx -\beta \ln \omega + \gamma, \tag{3}$$

where β is the slope used to classify the signal, $\|\widehat{X}_\omega\|$ is the modulus of the Fourier coefficients, and γ is a constant. Signals with the same PSD on average, and hence $\beta = 0$, are classified as white noise. Signals with a higher slope ratio are more trending. Two common examples of such trending signals are pink noise, where $\beta = 1$, and red noise, where $\beta = 2$.

There are various techniques available for replicating noise signals, including procedural noise techniques [4], Auto-Regressive (AR) models [11], and white noise filtering techniques [2]. Each technique has its advantages and disadvantages, which depend on factors such as computational complexity and similarity to known phenomena. In this work, we propose a novel technique that involves adjusting the scaling law, drawing inspiration from the squared-root spectrum [10]. Our approach adjusts the equation:

$$\widehat{X}_\omega = \eta(0,1)\omega^{-s\beta/2}, \tag{4}$$

where $\eta(0,1)$ are zero-mean Gaussian-distributed samples with unit standard deviation, and s is an adjustable parameter. To determine the optimal value of s, we consider the scaling law (β) of each slice 0D+1 of the noise to be generated. We then increment or decrement s to ensure that the set average of β equals the desired value. Compared to other techniques, our proposed method offers the unique advantage in that it allows for precise control over the generated noise's properties with many dimensions. By adjusting the scaling law, we can fine-tune the noise signal to match specific characteristics of real-world phenomena. Additionally, our approach has low computational complexity, making it suitable for a wide range of applications.

2 Metodology

We generate one and two-dimensional noise, employing the SRSM methodology. In this technique, we measure the PSD in the rows and columns of the matrix in an attempt to recover the same PSD used initially shown in Figure 2. It is important to emphasize that this methodology builds the signal in two dimensions and allows the conservation of structures at low frequencies, which can be visually perceptible, mainly for $\beta > 1$.

Then we analyze all these data using the `ordpy` algorithm proposed in Ref. [9] to calculate the entropy complexity described in the $H \times C$ manifold, where C represents the complexity and H is the normalized entropy. For this, we test different values of n and use only a delay value equal to 1.

Figure 5 shows the PSD in a log scale for white, pink, and red noise and the projection in the $H \times C$ space for a time series of 2^{15} points for emending dimension $n = 6$ reproducing the results shown in Ref. [9].

Then, we analyze the impact of the size of the time series in a different *embedding* dimension. We notice that the series size and the *embedding* play an essential role, as shown in Figure 2.

Then, we analyse the impact of different 2D colored noise in several *embedding* dimension configurations; cf. Figure 3

3 Results

The results indicated that the size of the time series is an important factor in the $H \times C$ manifold. In general, longer time series led to better separation of clusters and a clearer distinction between

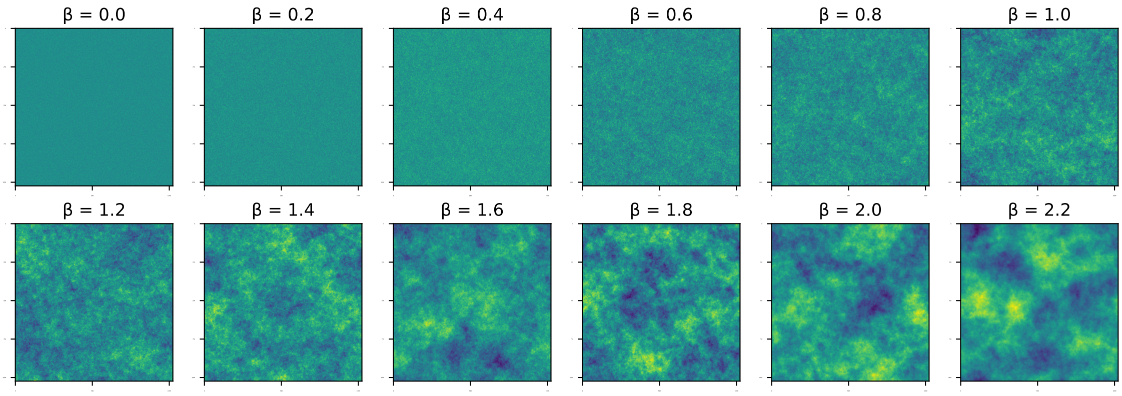

Figure 2: Two-dimension colored noise generated for 1024x1024 points for different values of index β. Larger values of β generate several structures.

different classes of data. This suggests that including more temporal information can improve entropy permutation analysis.

In addition, the *embedding* dimension also proved to be important in the characterization of the $H \times C$ space. For time series, smaller windows, e.g., 1024 to 2^{15} points, led to a clearer separation of clusters. However, for $1/f$ images, larger windows, e.g., 128×128 or 1024×1024 pixels, were needed to obtain meaningful results. This may be related to the complexity of the image and the need to include more detailed spatial information. The results showed that the three analyses occupied different regions in the $H \times C$ manifold. This suggests that the information contained in the different types of data is significantly different and that the points are not easily separable in $H \times C$ manifold.

This difficulty in separating the points can be a problem for feature extraction for deep learning analysis. If the points are not easily separable, it may be difficult to find discriminating features useful for classification tasks or pattern detection. A possible approach to overcome this difficulty is using of data pre-processing techniques, such as filtering and normalization. These techniques can help to reduce the variability between different types of data and, consequently, improve the separability of points in the $H \times C$ space.

4 Concluding Remarks

The permutation entropy analysis for colored noise data showed that the “original data", “signallers noise" and “signal only" do not occupy the same regions in the $H \times C$ space, making it difficult to extract features for deep learning analysis based on the PEn attributes. It is important to consider pre-processing techniques to improve the separability of points in $H \times C$ space and thus allow the extraction of discriminated features.

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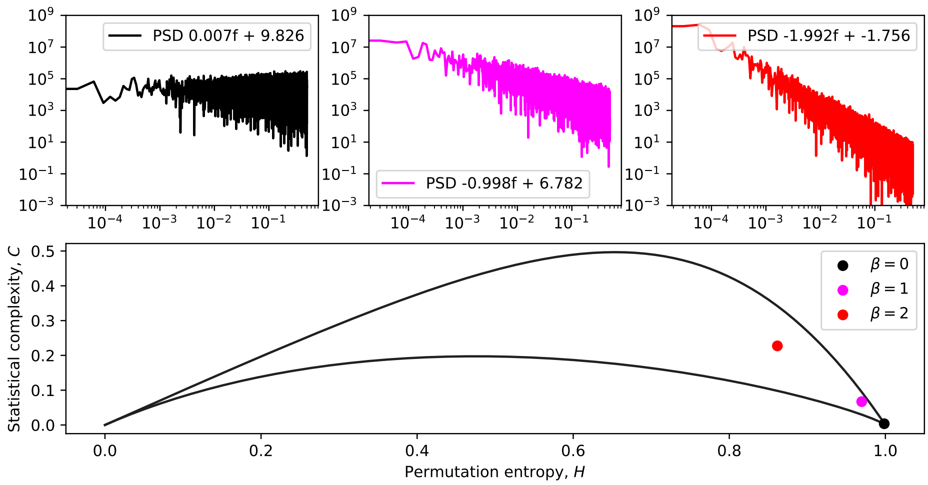

Figure 3: The position of $1/f^{\beta}$ time series in the $H \times C$ space. This figure show that our methodology reproduce the results for the `ordpy` paper [9].

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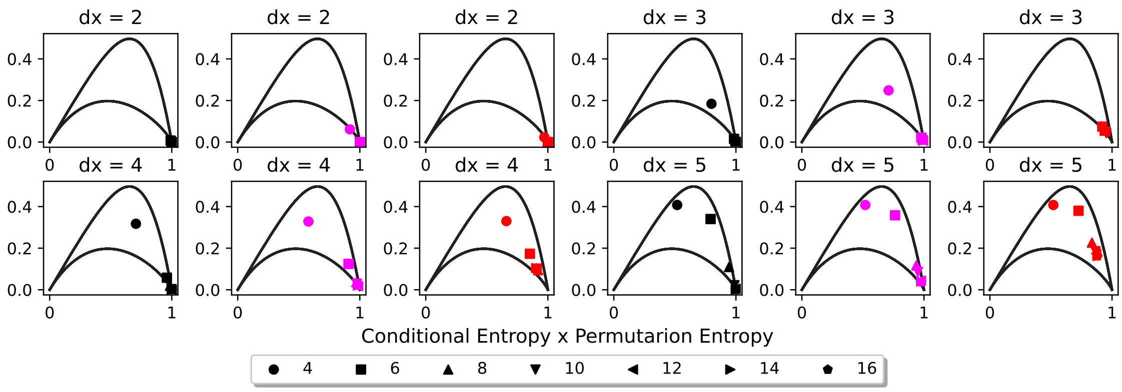

Figure 4: $H \times C$ space for different *embedding* dimensions ($d_x = 2, 3, 4$ and 5) for several kind of colored noise. White, magenta and red represent noise index spectral $\beta = 0, 1$ and 2 respectively.

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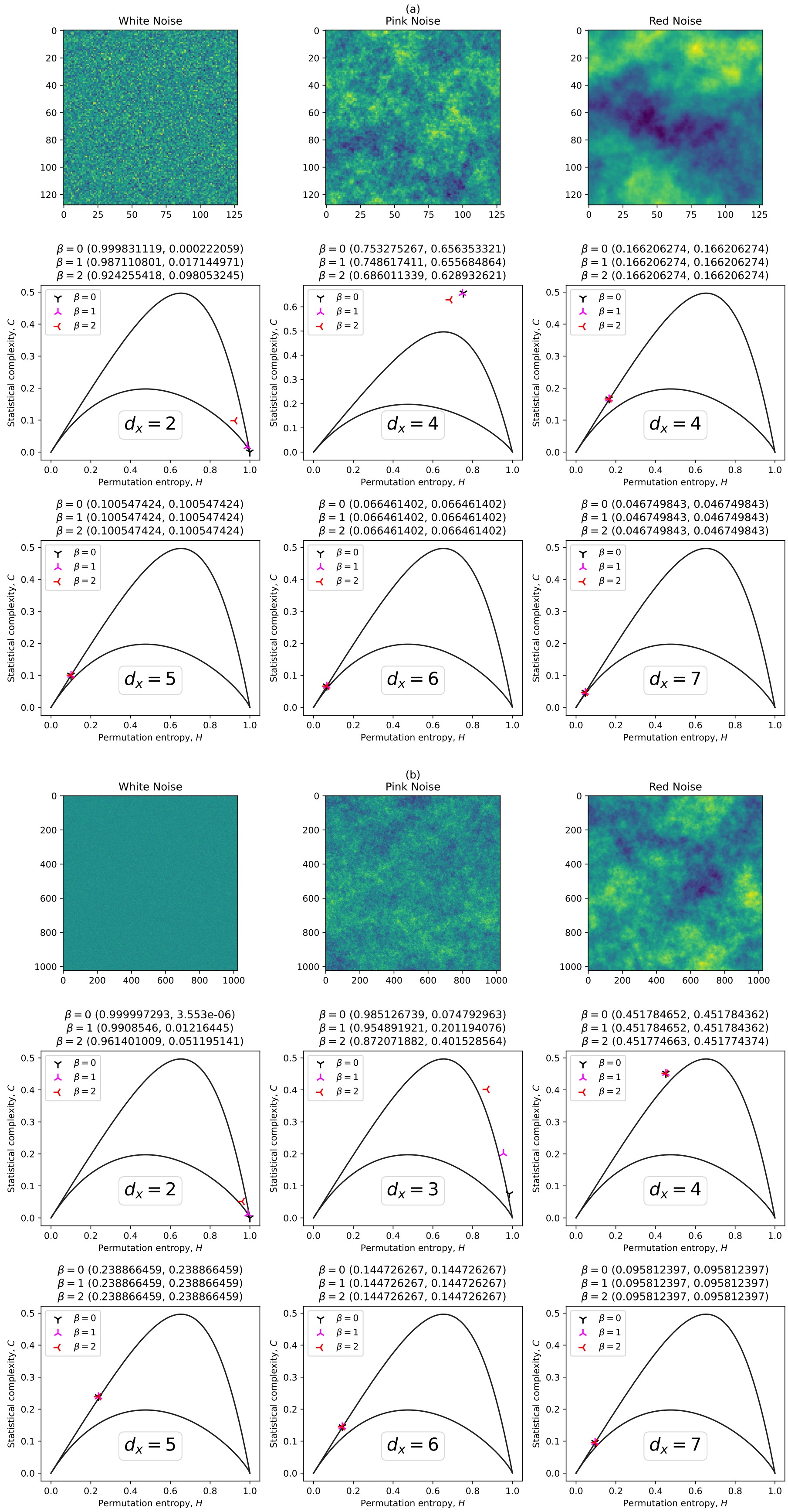

Figure 5: The first few lines in (a) and (b) represent the two-dimensional colored noise generated for 127x127 and 1024x1024 points. The second and third lines show the position in the $H \times C$ space of each of the images for different *embedding* dimensions. The title is the value of H and C respectively.