

Dark sector interaction and the supernova absolute magnitude tension

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It has been intensively discussed if modifications in the dynamics of the Universe at late times are able or not able to solve the H_0 tension. On the other hand, it has also been argued that the H_0 tension is actually a tension on the supernova absolute magnitude M_B . In this work, we robustly constrain M_B using Pantheon supernovae Ia sample, baryon acoustic oscillations, and big bang nucleosynthesis data, and assess the M_B tension by comparing three theoretical models, namely the standard Λ CDM, the w CDM, and a nongravitational interaction between dark energy (IDE) and dark matter. We find that the IDE model can solve the M_B tension with a coupling different from zero at 95% CL, confirming the results obtained using a H_0 prior.

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I. INTRODUCTION

The standard Λ -cold dark matter (Λ CDM) scenario provides a wonderful fit to astronomical observations carried out over the past two decades. On the other hand, recently some tensions and anomalies became statistically significant while analyzing different data sets, placing the Λ CDM cosmology at a crossroads. The most discussed and statistically significant tension in the literature is in the estimation of the Hubble constant, H_0 , between the cosmic microwave background (CMB) and the direct local distance ladder measurements. Assuming the Λ CDM scenario, Planck-CMB data analysis provides $H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [1], which is in 4.4σ tension with the local measurement $H_0 = 74.03 \pm 1.42 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [2] by the SH0ES team, or 4.2σ considering the latest one $H_0 = 73.2 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [3]. Additionally, many other late time measurements are in agreement with a higher value for the Hubble constant (see the discussion in [4–6]) and in tension with the Planck-CMB estimate, as for example the Megamaser Cosmology Project [7] that gives $H_0 = 73.9 \pm 3.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$, or using the surface brightness fluctuations [8] that find $H_0 = 73.3 \pm 0.7 \pm 2.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The lower value of H_0 inferred from the Planck-CMB data is in very good agreement with baryon acoustic oscillations (BAO) + big bang nucleosynthesis (BBN) constraints [9], and other CMB experiments like ACTPol-DR4 [10] or SPT-3G [11]. Finally, there are measurements in the middle that can not discriminate

between the two values, such as the tip of the Red Giant branch [12] or the astrophysical model dependent measurements based on the strong lensing effect [13]. Motivated by these observational discrepancies (unlikely to disappear completely by introducing multiple and unrelated systematic errors) it has been widely discussed in the literature whether new physics beyond the standard cosmological model can solve the H_0 tension (see Refs. [14–16] and references therein for a review).

The SH0ES team measures the absolute peak magnitude, M_B , of type Ia supernovae (SN), assumed to be standard candles, by calibrating the distances of SN host galaxies to local geometric distance anchors via the Cepheid period luminosity relation. The magnitude M_B is then converted into a value of H_0 via the magnitude-redshift relation of the Pantheon SN sample [17]. Therefore, it has been argued that the H_0 tension is actually a tension on the supernova absolute magnitude M_B [18,19], because the SH0ES H_0 measurement comes directly from M_B estimates. The CMB constraint on the sound horizon to the SN absolute magnitude M_B using the parametric-free inverse distance ladder predicts $M_B = -19.401 \pm 0.027 \text{ mag}$ [20], while the SN measurements from SH0ES corresponds to $M_B = -19.244 \pm 0.037 \text{ mag}$ [19]. These measurements are at 3.4σ tension. Thus, as argued in [18,19], rather than explaining the H_0 tension one should instead focus on the supernova absolute magnitude tension, because this is what the Cepheid calibrations are designed to measure. For this reason, it has been shown that modifications of the expansion history at late times can not help with the H_0 and/or M_B tension, motivating discussions and investigations in this direction [21–27]. On the other hand, other

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possibilities, including early time solutions that introduce new physics prior to recombination, also have been considered as good alternatives to solve the H_0 tension (see [28–47] and reference therein), even if they are not completely successful [48,49].

In particular, alternative scenarios involving a nongravitational interaction between the two main dark species of our Universe, namely, dark matter (DM) and dark energy (DE), have been intensively studied as a possibility to resolve the current cosmological H_0 tension (see for example [6,50–77] for a short list and references therein). This class of models has been listed as an example of a “not working” solution, i.e., able to solve the H_0 tension, but unable to alleviate the M_B tension. Therefore, we aim with this paper to check if modifications to the late time evolution involving a non-gravitational coupling different from zero in the dark sector, can bring the M_B measured from distant supernovae of the Pantheon sample and calibrated by BAO + BBN into agreement with the value obtained calibrating the local SN with the Cepheid measurements. We are interested in the background only analysis, so we don’t include CMB data in this paper. But, without loss of generality, using the mentioned data, we will get good accuracy in our constraints. Our main conclusion is that an Interacting DE-DM model (IDE) with a dark coupling different from zero is able to solve the $H_0 - M_B$ tension, and it is in agreement with the Pantheon SN sample.

This paper is structured as follows. In Sec. II we introduce the interacting DE-DM model considered. In Sec. III we present the data sets and methodology used in this work. In Sec. IV we discuss the main results of our analysis. In Sec. V we outline our final considerations and perspectives.

II. THEORETICAL FRAMEWORK

In this section we introduce the interacting DE-DM model studied here. In a homogeneous and isotropic Universe, the dark interaction is quantified as

$$\nabla_\mu T_i^{\mu\nu} = Q_i^\nu, \quad \sum_i Q_i^\mu = 0, \quad (1)$$

where the index i runs over DM and DE. The four-vector Q_i^μ governs the interaction. We can assume that Q_i^μ is given by

$$Q_i^\mu = (Q_i + \delta Q_i)u^\mu + a^{-1}(0, \partial^\mu f_i), \quad (2)$$

where u^μ is the velocity four-vector and Q_i is the background energy transfer. Let us note that from now on we shall use the notation $Q_i \equiv Q$. The symbol f_i refers to the momentum transfer potential. In the Friedmann-Lemaître-Robertson-Walker (FLRW) background, one can write down the conservation equations of the DM and DE densities as

$$\dot{\rho}_c + 3H\rho_c = Q, \quad (3)$$

$$\dot{\rho}_x + 3H(1+w)\rho_x = -Q, \quad (4)$$

where $H = \dot{a}/a$ is the expansion rate of the Universe and w is the equation of state (EoS) of DE.

In the present work, we consider a very well-known parametric form of the interaction function Q , namely, $Q = \mathcal{H}\xi\rho_x$, where ξ is the coupling parameter between the dark components. From the sign of ξ , one can identify the direction of the energy flow between the dark sectors. The condition $\xi < 0$ corresponds to the energy flow from DM to DE, and $\xi > 0$ represents the opposite scenario. The functional form $Q = \mathcal{H}\xi\rho_x$ can avoid the instabilities in the perturbations at early times on the dark sector species, although it will not be necessary to consider the evolution of the perturbations in this work. It should be noted that the inclusion of the global factor \mathcal{H} into the interaction function Q was motivated to quantify a possible global interaction through cosmic history. Although, as already argued by the authors [78], the interaction in the dark sectors should depend on the local quantities too. Nevertheless, as the present interaction function is widely studied in the literature, we aim to revisit this model.

On the other hand, the choice of the interaction function is not unique and it is very difficult to provide a specific functional form since the nature/properties of both dark components is completely unknown at present. From the observational perspective, we do not find any strong signal which could reveal the nature of the dark components, and hence, we believe that the nature of the interaction function will probably remain unknown for the next decade(s). We can only approximate the interaction function Q through theoretical arguments and the consistency with observational data. However, it has been argued by many investigators that the interaction between the dark components may appear from some effective field theory [79], disformal coupling [80], axion monodromies [81], varying dark matter mass and fundamental constants [82], and Horndeski theories [83]. Thus, one can see that some viable formalism can be given for the interacting dark energy theory. Motivated to check the strength of this dark coupling and its observational viability, let us assume in the present work the simple parametric function $Q = \mathcal{H}\xi\rho_x$ to quantify such dark interaction.

III. DATASETS AND METHODOLOGY

In this section we present the data sets and methodology used to obtain the observational constraints on the model parameters by performing Bayesian Monte Carlo Markov Chain (MCMC) analysis. In order to constrain the parameters, we use the following data sets:

- (a) Type Ia Supernovae distance moduli measurements from the Pantheon sample [17]. These measurements constrain the uncalibrated luminosity distance $H_0 d_L(z)$,

or in other words the slope of the late-time expansion rate (which in turn constrains Ω_m). We refer to this dataset as Pantheon. For a SN at redshift z , the theoretical apparent magnitude m_B is given by

$$m_B = 5 \log_{10} \left[\frac{d_L(z)}{1 \text{ Mpc}} \right] + 25 + M_B, \quad (5)$$

where M_B is the absolute magnitude. The distance modulus is then written as $\mu(z) = m_B - M_B$.

- (b) Baryon acoustic oscillations (BAO) distance and expansion rate measurements from the 6dFGS [84], SDSS-DR7 MGS [85], BOSS DR12 [86] galaxy surveys, as well as from eBOSS DR14 Lyman- α ($\text{Ly}\alpha$) absorption [87] and $\text{Ly}\alpha$ -quasars cross correlation [88]. These consist of isotropic BAO measurements of $D_V(z)/r_d$ (with $D_V(z)$ and r_d the spherically averaged volume distance, and sound horizon at baryon drag respectively) for 6dFGS and MGS, and anisotropic BAO measurements of $D_M(z)/r_d$ and $D_H(z)/r_d$ (with $D_M(z)$ the comoving angular diameter distance and $D_H(z) = c/H(z)$ the Hubble distance) for BOSS DR12, eBOSS DR14 $\text{Ly}\alpha$, and eBOSS DR14 $\text{Ly}\alpha$ -quasars cross correlation.
- (c) The state of the art assumptions on big bang nucleosynthesis (BBN). The BBN data consist of measurements of the primordial abundances of helium, Y_p , from [89], and the deuterium measurement, $y_{DP} = 10^5 n_D/n_H$, obtained in [90]. This BBN likelihood is sensitive to the constraints of the physical baryon density $\omega_b \equiv \Omega_b h^2$ and the effective number of neutrino species N_{eff} . Let us fix $N_{\text{eff}} = 3.046$ in this present work. A Gaussian prior on $M_B = -19.244 \pm 0.037$ mag [19], corresponding to the SN measurements from SHOES.

We first consider the standard Λ CDM model, spanned by the following parameters: the Hubble constant H_0 , the physical baryon density $\omega_b \equiv \Omega_b h^2$, the physical cold dark matter density $\omega_c \equiv \Omega_c h^2$, and the absolute peak magnitude, M_B of SN. The matter density parameter today Ω_m is given by $\Omega_m = (\omega_b + \omega_c)/h^2$. Then, we consider two one-parameter extensions of the previous model. Firstly, we introduce a dark energy equation of state free to vary w and we call this model w CDM. Secondly, we consider a dark coupling between DM and DE with a single free parameter

ξ , and we refer to this extended scenario as the IDE model.

It should be noticed here that in order to avoid the gravitational instabilities present when $w = -1$ [91,92], we fix the dark energy equation of state to $w = -0.999$, as previously done in [59,93,94], without affecting the results, as shown by using simulated data in [61]. Moreover, in order to avoid early-time instabilities [91,92,95–98], we also assume $\xi < 0$.

We use the Metropolis-Hastings mode in CLASS + MontePython code [99–101] to derive the constraints on cosmological parameters from the data sets described above in this section, ensuring a Gelman-Rubin convergence criterion of $R - 1 < 10^{-3}$ [102].

IV. RESULTS

In this work, we consider two different data combinations, namely, Pantheon + BAO + BBN and Pantheon + BAO + BBN + M_B applied on three different cosmological models, namely, Λ CDM, w CDM, and IDE models.

The first joint analysis, shown in Table I, is chosen to break any possible degeneracy in the $H_0 - \Omega_m$ plane, because we know that BAO + BBN can constrain these parameters very well. Also, several authors have pointed out the ability of combined BAO and BBN data to probe the background cosmological history, independently of CMB data. As a second step for the analysis, we will add a M_B Gaussian prior corresponding to the SN measurements from SHOES, and we will analyze Pantheon + BAO + BBN + M_B , showing the results in Table II. In this work, we will analyze our baseline regardless of Planck-CMB data and then discuss the potential of the model of solving and/or alleviating the M_B tension.

Figure 1 shows the 2D-joint posterior distributions at 68% CL and 95% CL in the $M_B - H_0$ plane, obtained from Pantheon + BAO + BBN and Pantheon + BAO + BBN + M_B for the three scenarios considered in this work. In particular we compare Λ CDM in the top left panel of Fig. 1, the w CDM framework in the top right panel of Fig. 1, and the IDE scenario in the bottom panel of Fig. 1.

Once we have obtained constraints on the above cosmological parameters, we can quantify the level of concordance or discordance (if any) on M_B between different datasets i and j using a simple 1D tension metric as follows:

TABLE I. Constraints at 95% CL for the four models under consideration in this work for the Pantheon + BAO + BBN joint analysis. The parameter H_0 is measured in units of $\text{km s}^{-1} \text{Mpc}^{-1}$ and M_B in units of mag. The last column represents the degree of tension with M_B from SHOES.

Model	M_B	H_0	Ω_m	w	ξ	Tension
Λ CDM	$-19.393^{+0.042}_{-0.040}$	$68.5^{+1.2}_{-1.1}$	$0.302^{+0.025}_{-0.025}$	-1	0	3.4σ
w CDM	$-19.376^{+0.095}_{-0.11}$	$69.2^{+3.3}_{-3.7}$	$0.306^{+0.032}_{-0.034}$	$-1.02^{+0.11}_{-0.11}$	0	2.1σ
IDE	$-19.385^{+0.094}_{-0.089}$	$69.1^{+3.1}_{-2.8}$	$0.274^{+0.044}_{-0.050}$	-0.999	> -0.35	2.4σ

TABLE II. Constraints at 95% CL for the four models under consideration in this work for the Pantheon + BAO + BBN + M_B joint analysis. The parameter H_0 is measured in units of $\text{km s}^{-1} \text{Mpc}^{-1}$ and M_B in units of mag. The last column represents the degree of tension with M_B from SHOES.

Model	M_B	H_0	Ω_m	w	ξ	Tension
ΛCDM	$-19.340^{+0.047}_{-0.046}$	$70.0^{+1.4}_{-1.4}$	$0.314^{+0.024}_{-0.025}$	-1	0	2.1σ
$w\text{CDM}$	$-19.285^{+0.049}_{-0.060}$	$72.4^{+1.9}_{-2.2}$	$0.323^{+0.028}_{-0.025}$	$-1.10^{+0.09}_{-0.10}$	0	1.0σ
IDE	$-19.288^{+0.064}_{-0.063}$	$72.3^{+2.4}_{-2.3}$	$0.256^{+0.061}_{-0.065}$	-0.999	$-0.31^{+0.27}_{-0.28}$	0.9σ

$$T_{M_B} \equiv \frac{M_{B,i} - M_{B,j}}{\sqrt{\sigma_{M_{B,i}}^2 + \sigma_{M_{B,j}}^2}}, \quad (6)$$

where the value of T_{M_B} can directly be interpreted as the level of tension in equivalent Gaussian σ .

The first thing to notice is that (taking as a reference the SN measurements from the SHOES team obtained by Ref. [19], $M_B = -19.244 \pm 0.037$) our constraints on M_B are in tension above 3σ for the ΛCDM model.

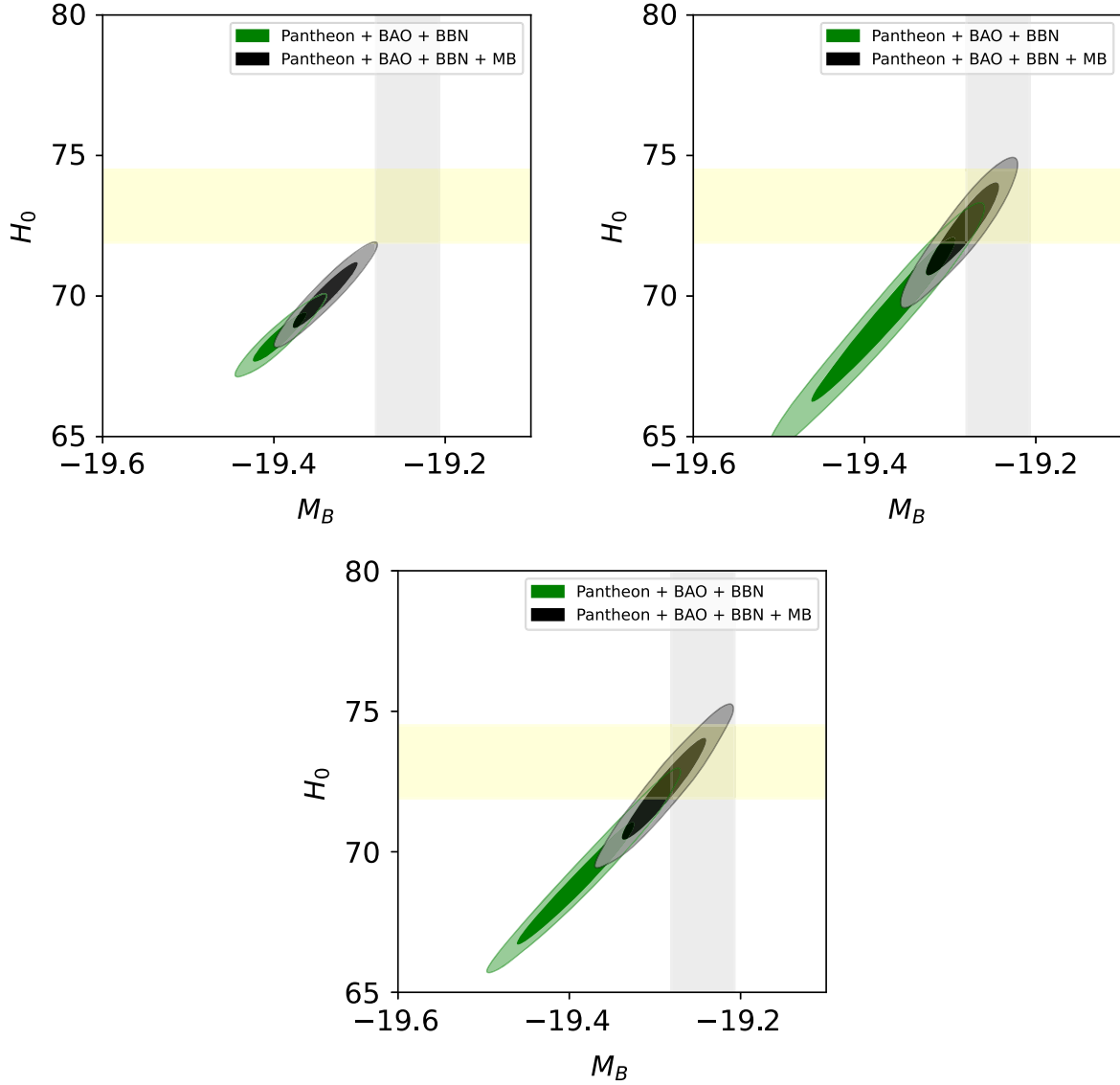


FIG. 1. Two-dimensional joint posterior distributions in the M_B - H_0 plane, with the corresponding 68% CL and 95% CL contours, obtained for Pantheon + BAO + BBN and Pantheon + BAO + BBN + M_B joint analyses within the ΛCDM (top left panel), $w\text{CDM}$ (top right panel), and IDE (bottom panel) models. The vertical gray band denotes the 68% CL interval of the SN absolute magnitude M_B based on the SHOES measurements. The horizontal yellow band denotes the 68% CL interval of the H_0 parameter based on the SHOES measurements.

On the contrary, both the w CDM and IDE models can reduce the M_B tension below 3σ . This shows that a late-time modification such as a phantom dark energy equation of state can alleviate the M_B tension as suggested when a H_0 prior is used.

Regarding the IDE scenario instead, the constraining power of the dataset combinations we are using is not enough to rule out the value of the coupling found from the works in the literature to alleviate the Hubble tension [6,50–75]. Actually, the dark-coupling estimate obtained in Table I has just a lower limit and it is in agreement with both zero and the large coupling needed to solve the H_0 tension, within one standard deviation, because of the large error bars.

Considering the Pantheon + BAO + BBN combination (Table I), we can see that the introduction of a non-null interaction between DM and DE shifts slightly both H_0 towards higher values and M_B towards lower values, relaxing the error bars and reducing the tension with SHOES below 3σ (see also Fig. 1, bottom panel).

If we now add a Gaussian prior M_B to the previous dataset combination, we obtain the results in Table II. Here we can see that, even if in disagreement with the Λ CDM model at more than 3σ , the addition of the M_B prior can shift its value reducing the tension below 3σ . On the contrary, for the w CDM model, the combination with the Gaussian M_B prior solves the tensions with the SHOES measurement thanks to the evidence for a phantom dark energy at more than 95% CL. Moreover, for the IDE model the M_B prior can be included safely, restoring the complete agreement with the SHOES data at the price of a negative coupling $\xi = -0.31^{+0.27}_{-0.28}$ at 95% CL. This result confirms the claim we find in the literature [6,50–75], i.e., that a flux of energy from the DM to the DE can solve the Hubble tension, and show that the use of a Gaussian prior on H_0 or M_B is equivalent for this class of models.

To conclude the analysis, we plot the magnitude-redshift relation for the Pantheon sample in the redshift range $0 < z < 1$, together with the best fits obtained for Pantheon + BAO + BBN + M_B for a Λ CDM model (blue line), the IDE model (orange line) and the w CDM scenario (green line) in the left panel of Fig. 2. We show instead the $100 \times (1 - m_B^i/m_B^{\Lambda\text{CDM}})$, where i runs on the w CDM and IDE models from the difference of the best fit values between Λ CDM and w CDM scenarios (blue line), or Λ CDM and IDE models (orange line) in the right panel of Fig. 2, i.e., the percentage difference between the best fits of the models with respect to the Λ CDM case. We find that in the z range adopted in SHOES SN data analysis, the difference between the IDE scenario and the Λ CDM one is $<1.4\%$, decreasing up to $<0.6\%$ at high z . For the w CDM model, we note practically the same difference at very low z ($<1.35\%$), and $<0.5\%$ at high z .

To better quantify the agreement (and/or disagreement) between the models and the Pantheon SN sample, we perform a statistical comparison of the IDE model with the Λ CDM scenario by using the well-known Akaike information criterion (AIC) [103].

The AIC is defined through the relation

$$\text{AIC} \equiv -2 \ln \mathcal{L}_{\text{max}} + 2N, \quad (7)$$

where \mathcal{L}_{max} is the maximum likelihood function of the model, and N is the total number of free parameters in the model. For the statistical comparison, the AIC difference between the model under study and the reference model is calculated for the joint analysis Pantheon + BAO + BBN + M_B . This difference in AIC values can be interpreted as the evidence in favor of the model under study over the reference model. It has been argued in [104] that one model can be preferred with respect to another if the AIC difference between the two models is greater than a threshold value $\Delta_{\text{threshold}}$. As a rule of thumb, $\Delta_{\text{threshold}} = 5$

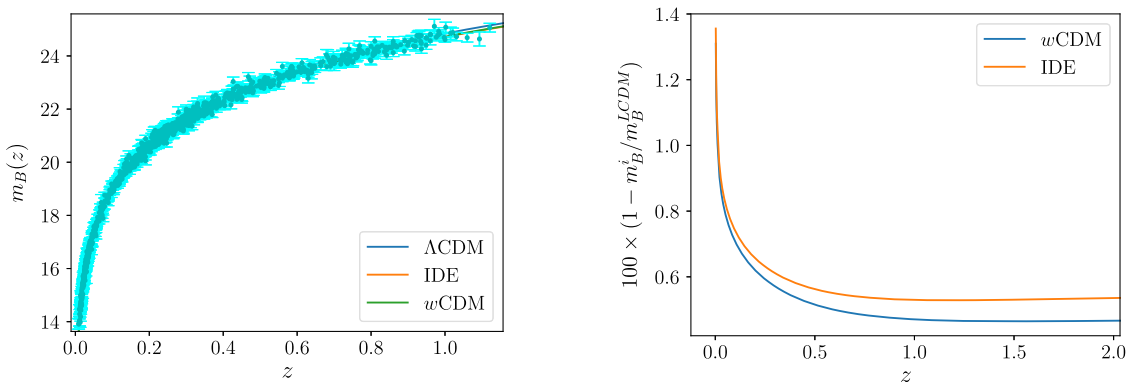


FIG. 2. Left panel: Best fit values for the Λ CDM, w CDM and IDE models obtained by Pantheon + BAO + BBN + M_B compared against the magnitude-redshift relation of the Pantheon SN sample in the redshift range $0 < z < 1$. Right panel: $100 \times (1 - m_B^i/m_B^{\Lambda\text{CDM}})$, where i run on the w CDM and IDE models from the difference of the best fit values between Λ CDM and w CDM or IDE of the left plot.

can be considered the minimum value to assert a strong support in favor of the model with a smaller AIC value, regardless of the properties of the models under comparison [105].

The result we obtain by computing $AIC^{\Lambda\text{CDM}} - AIC^{\text{IDE}} = \Delta AIC = 1.54$, which shows that the IDE model can better fit the joint analysis than the ΛCDM model, but the ΛCDM framework cannot be ruled out with high statistical significance. Actually, a simple χ^2 comparison also points in the same direction. We find for the fit of Pantheon + BAO + BBN + M_B a $\chi^2_{\min} = 1046.74$ for the IDE model, and $\chi^2_{\min} = 1050.28$ for the ΛCDM model, considering the same joint analysis, corresponding to a $\Delta\chi^2_{\min} = -3.57$ for one additional degree of freedom. This shows that the IDE model can fit these data better than the ΛCDM scenario.

A similar result is obtained for the $w\text{CDM}$ models, that gives $AIC^{\Lambda\text{CDM}} - AIC^{w\text{CDM}} = \Delta AIC = 1.52$, improving the fit of the joint analysis with respect to the standard ΛCDM , even if not in a significant way. Moreover, the $\chi^2_{\min} = 1046.76$ for the $w\text{CDM}$ model, with a $\Delta\chi^2_{\min} = -3.52$ and only one more degree of freedom.

Therefore, even if the ΛCDM model fits slightly better to the SN data, the best fit for the IDE case, which can solve the Hubble tension with a 95% CL of evidence for a dark coupling different from zero, cannot be excluded by the Pantheon + BAO + BBN + M_B combination of data. In other words, for this class of interacting models, solving the H_0 or the M_B tension is equivalent, and using a Gaussian prior from SH0ES on the Hubble constant does not bias the results. Actually, the constraints obtained are completely in agreement with the magnitude-redshift relation of the Pantheon SN sample.

V. FINAL REMARKS

It has been intensively discussed in [18,19] if modifications in the dynamics of the Universe at late times is able or not to solve the H_0 tension once BAO and Pantheon data are taken into account. On the other hand, it has also been

argued that the H_0 tension is actually a tension on the supernova absolute magnitude M_B .

We investigate this issue by analyzing three different models, namely the ΛCDM scenario and two possible late-time solutions to the Hubble tension, i.e., a $w\text{CDM}$ model and an interacting DE-DM model. We analyze these scenarios making use of two possible dataset combinations, that are Pantheon + BAO + BBN and Pantheon + BAO + BBN + M_B . We find that the IDE model can alleviate the M_B tension, as well as the H_0 disagreement, and a coupling different from zero at more than 95% CL is actually preferred by the Pantheon + BAO + BBN + M_B dataset combination. The same thing is also true for a phantom solution with $w < -1$, because it can solve the M_B tension, as it does for the H_0 tension.

As a final step we investigate if the best fit from an interaction between the DM and the DE for the Pantheon + BAO + BBN + M_B data, than can solve the Hubble tension, is in agreement with the Pantheon data and we find that this model is preferred against the ΛCDM case. This result allows us to conclude that applying a SH0ES H_0 prior, instead of a M_B prior, does not bias the results for an IDE late time solution. On the contrary, if we repeat the same analysis for a phantom DE equation of state, we do not see an indication for this solution from Pantheon data only, that prefers in any case, a cosmological constant.

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