

Reconstruction of the dark sectors' interaction: A model-independent inference and forecast from GW standard sirens

Alexander Bonilla ¹★, Suresh Kumar,^{2,3}★ Rafael C. Nunes⁴★ and Supriya Pan⁵

¹*Departamento de Física, Universidade Federal de Juiz de Fora, 36036-330 Juiz de Fora, MG, Brazil*

²*Department of Mathematics, Indira Gandhi University, Meerpur, Haryana 122502, India*

³*Department of Mathematics, National Institute of Technology, Kurukshetra, Haryana 136119, India*

⁴*Divisão de Astrofísica, Instituto Nacional de Pesquisas Espaciais, Avenida dos Astronautas 1758, São José dos Campos, 12227-010 SP, Brazil*

⁵*Department of Mathematics, Presidency University, 86/1 College Street, Kolkata 700073, India*

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ABSTRACT

Interacting dark matter (DM) – dark energy (DE) models have been intensively investigated in the literature for their ability to fit various data sets as well as to explain some observational tensions persisting within the Λ CDM cosmology. In this work, we employ the Gaussian processes (GP) algorithm to perform a joint analysis by using the geometrical cosmological probes such as Cosmic chronometers, Supernova Type Ia, Baryon Acoustic Oscillations, and the H0LiCOW lenses sample to infer a reconstruction of the coupling function between the dark components in a general framework, where the DE can assume a dynamical character via its equation of state. In addition to the joint analysis with these data, we simulate a catalogue with standard siren events from binary neutron star mergers, within the sensitivity predicted by the Einstein Telescope, to reconstruct the dark sector coupling with more accuracy in a robust way. We find that the particular case, where $w = -1$ is fixed on the DE nature, has a statistical preference for an interaction in the dark sector at late times. In the general case, where $w(z)$ is analysed, we find no evidence for such dark coupling, and the predictions are compatible with the Λ CDM paradigm. When the mock events of the standard sirens are considered to improve the kernel in GP predictions, we find a preference for an interaction in the dark sector at late times.

Key words: cosmological parameters – cosmology: observations – dark energy – dark matter.

1 INTRODUCTION

Up-to-date observational data suggest that our Universe is mainly driven by a pressure-less or cold dark matter (CDM) and a dark energy (DE) fluid where around 96 per cent (~ 28 per cent DM + 68 per cent DE) of the total energy budget of the universe is occupied by this joint dark fluid (Planck Collaboration VI 2020). The fundamental nature of these fluids, such as, their origin and dynamics are yet to be known even after a series of astronomical missions. Therefore, understanding the dark picture of the universe has remained one of the greatest challenges in cosmology. In order to reveal the physics of the dark sectors, various cosmological models have been proposed and investigated over the last several years (Copeland, Sami & Tsujikawa 2006; Cai et al. 2010; De Felice & Tsujikawa 2010; Sotiriou & Faraoni 2010; Capozziello & De Laurentis 2011; Bamba et al. 2012; Clifton et al. 2012; Cai et al. 2016; Nojiri, Odintsov & Oikonomou 2017; Bahamonde et al. 2021). The standard cosmological model Λ CDM is one of the simplest cosmological models which fits excellently to most of the observational probes. However, the physics of the dark fluids is not clear in this model as well – the cosmological constant problem, for instance, is a serious issue (Weinberg 1998). Additionally, in this canonical picture of the universe, several anomalies and tensions

between different cosmological probes may indicate a revision of the Λ CDM cosmology (see refs. Perivolaropoulos & Skara 2021; Schöneberg et al. 2021). In the Λ CDM model, we assume the simplest possibility for its ingredients – the independent evolution of DM and DE. As the physics of the dark sector is not yet clear, there should not be any reason to exclude the possibility of an interaction between these components. By allowing the interaction or the energy exchange between DM and DE, one naturally generalizes the non-interacting scenarios.

The theory of the dark sector interaction did not appear suddenly in the literature. The limitations or issues of the standard cosmological model at the fundamental level motivated to relax the independent evolution of DM and DE. For instance, an interaction in the dark sector can provide a possible/promising explanation/solution to the cosmic coincidence problem (del Campo, Herrera & Pavón 2009; Velten, vom Martens & Zimdahl 2014), H_0 tension (Di Valentino, Melchiorri & Mena 2017; Kumar & Nunes 2017; Yang et al. 2018; Pan et al. 2019; Di Valentino et al. 2020; Lucca & Hooper 2020; Akarsu et al. 2021; Anchordoqui et al. 2021; Di Valentino 2021; Di Valentino et al. 2021; Kumar 2021; Renzi, Hogg & Giarè 2021; Theodoropoulos & Perivolaropoulos 2021), that arises between the CMB measurements by *Planck* satellite within the Λ CDM cosmology (Planck Collaboration VI 2020) and SH0ES (Riess et al. 2019, 2021), and S_8 tension (Poursidou & Tram 2016; An, Feng & Wang 2018; Kumar, Nunes & Yadav 2019; de Araujo et al. 2021; Lucca 2021; Avila et al. 2022) that arises between the *Planck* and weak lensing measurements (Di Valentino et al. 2021). Additionally, an

* E-mail: alex.acidjazz@gmail.com (AB); rafadcnunes@gmail.com (RN); sukuyd@gmail.com (SK)

interaction in the dark sector could explain the phantom phase of the DE without any need of a scalar field with negative correction (Wang, Gong & Abdalla 2005; Mohseni Sadjadi & Honardoost 2007; Pan & Chakraborty 2014; Bonilla Rivera & García Farieta 2016; Bonilla & Castillo 2018; Yang et al. 2021). See Bolotin et al. (2015), Wang et al. (2016) for a comprehensive reading on interacting dark energy models. Therefore, based on such appealing outcomes, it is indeed desirable to consider a wider picture of our Universe by including the interaction between DM and DE, and allow the observational data to favour or reject this possibility.

In a standard approach, the interaction between DM and DE is investigated through the inclusion of some phenomenological coupling function to describe the DM and DE dynamics intuitively. However, let us recall that some action formalisms, i.e. construction of the DE–DM interaction models from the first principle, including the Noether symmetry approach, have also been developed in the literature, see e.g. Piedipalumbo, De Laurentis & Capozziello (2020), Gleyzes et al. (2015), Böhmer, Tamanini & Wright (2015), D’Amico, Hamill & Kaloper (2016), Kase & Tsujikawa (2020), Vanrietvelde et al. (2020), Pan, Sharov & Yang (2020). On the other hand, given the great interest of the community in this theoretical framework, accomplishing a model independent analysis becomes a necessary task. In principle, one may do it using cosmographic approach, wherein a series expansion is performed around $z = 0$ for a cosmological observable, and then the data are used to constrain the kinematic parameters. This procedure works fine for lower values of z , but may not be good enough for larger values of z (see Lorenz et al. 2021). An interesting and robust alternative could be to consider a Gaussian process (GP) to reconstruct the cosmological parameters in a model-independent way (Seikel, Clarkson & Smith 2012; Shafieloo, Kim & Linder 2012; Zhang & Xia 2016; Wang & Meng 2017; Gómez-Valent & Amendola 2018; Liao et al. 2019; Bengaly, Clarkson & Maartens 2020; Jesus et al. 2020; Nunes et al. 2020; Renzi & Silvestri 2020; Bernardo et al. 2021; Colgáin & Sheikh-Jabbari 2021; Dialektopoulos et al. 2022; Mehrabi & Rezaei 2021; Sun, Jiao & Zhang 2021; Avila et al. 2022; Bengaly 2022), or to fix a class of cosmological models (Briffa et al. 2020; Bernardo & Levi Said 2021; Elizalde, Gluza & Khurshudyan 2021; Reyes & Escamilla-Rivera 2021; Ren et al. 2021). The GP and other alternative approaches have been applied to reconstruct an interaction between DM and DE in a minimally model-dependent way in various works with different data sets and approximations (Wang et al. 2015; Yang, Guo & Cai 2015; Cai, Tamanini & Yang 2017; Mukherjee & Banerjee 2017; Martinelli et al. 2019; von Martens et al. 2019; Zhou, Zhang & Li 2019; Yang 2020; von Martens et al. 2021).

In this work, we employ the GP to carry out a joint analysis by using some geometrical cosmological probes, viz., Cosmic chronometers (CC), Supernova Type Ia (SN), Baryon Acoustic Oscillations (BAO), and the H0LiCOW lenses sample to constrain/reconstruct the interaction in the dark sector of the universe in two different frameworks, namely, the one where the EoS of DE mimics the vacuum energy (known as an interacting vacuum energy scenario) and secondly a general coupling scenario where DE is allowed to assume a dynamical character via its equation of state (EoS). This latter possibility has not been studied much in the literature, viz., most of the works are carried out with only the constant or linear approximation of the EoS parameter of DE. Moreover, to our knowledge of the current literature, the reconstruction of the interaction in the dark sector has not been performed using a joint analysis. In addition, we also simulate a catalogue of 1000 standard siren events from binary neutron star mergers, within the sensitivity predicted for the third generation of the ground GW detector called

the Einstein Telescope (ET), and we use these mock data to improve the reconstruction of the coupling function from the SN, BAO, CC, and H0LiCOW data. A model-independent joint analysis from above-mentioned data sets, including a forecast analysis with the simulated data for optimizing the covariance function (or kernel in GP language), as we present here, to our knowledge is new and not previously investigated in the literature. Indeed, a joint analysis with several observational probes is helpful to obtain tight constraints on the cosmological parameters. In this work, we develop this methodology to obtain an accurate and robust reconstruction of a possible interaction between DM and DE.

The paper is structured as follows. In Section 2, we describe the GP, the observational data sets, and the theoretical framework used in this work for model-independent inference of the dark sector coupling. In Section 3, we present and discuss our results on the reconstruction of the coupling function between DM and DE following the model-independent approach, wherein the Sections 3.1 and 3.2 describe two different reconstructed scenarios. Further, in Section 4, we use the mock gravitational waves data in order to get a more deeper understanding on the evolution of the coupling function. Finally, in Section 5, we conclude our work with a brief summary of the entire study.

2 METHODOLOGY, DATA SETS, AND THE THEORETICAL BACKGROUND

This section is divided into the following three parts: the Gaussian process, the observational data, and a basic framework of the theory, which we are going to test in this article using the observational data following the model-independent Gaussian approach.

2.1 Gaussian process

In a nutshell, the GP in cosmology allows us, given an observational data set $f(z) \pm \sigma_f$, to obtain a function $f(z)$ without the need to assume a parametrization or physical model about the dark nature of the main components of the universe. The GP method adequately describes the observed data based on a distribution over functions. The reconstructed function $f(z)$ (and its derivatives $f'(z)$, $f''(z)$, ..., etc) have a Gaussian distribution with mean and Gaussian error at each data point z . The functions at different points z and z' are related by a covariance function $k(z, z')$, which only depends on a set of kernels with hyperparameters l and σ_f , describing the strength and extent of the correlations among the reconstructed data points, respectively. Thus, l gives a measure of the coherence length of the correlation in the x -direction, and σ_f denotes the overall amplitude of the correlation in the y -direction. In general, the hyperparameters are constant since their values point to a good fit of the function rather than a model that mimics this behaviour, which means that the GP optimizes both concerning the observed data. Finally, the GP method is model-independent in a physical model and assumes a particular statistical kernel that determines the correlation between the reconstructed data points. The entire methodology used in this work is described in detail in section II of Ref. Bonilla, Kumar & Nunes (2021).

2.2 Observational data sets

In this section, we shall describe the geometrical probes in detail that we have used to trace the interaction in the dark sector.

(i) Cosmic Chronometers (CC): The CC approach is very powerful to detect the expansion history of the universe that comes through the measurements of the Hubble parameter. Here we take into

consideration 30 measurements of the Hubble parameter distributed over a redshift interval $0 < z < 2$ as in Ref. Moresco et al. (2016).

(ii) Supernovae Type Ia (SNs): The first astronomical data probing the accelerating expansion of our Universe are SNs. Certainly, SNs are very important astronomical probes in analysing the properties of DE and the expansion history of the universe. The latest compilation of SN data (Pantheon sample) that we have used in this work, consists of 1048 SN data points in the redshift range $0.01 < z < 2.3$ (Scolnic et al. 2018). In the context of a universe with zero curvature, the entire Pantheon sample can be summarized in terms of six model-independent $E(z)^{-1}$ data points (Riess et al. 2018). Here we use the six data points as reported in ref. Haridasu et al. (2018) in the form of $E(z)$ taking into account the theoretical and statistical considerations for its implementation.

(iii) Baryon Acoustic Oscillations (BAO): Another important cosmological probe is the BAO data. With the use of BAO, the expanding spherical wave produced by baryonic perturbations of acoustic oscillations in the recombination epoch can be traced through the correlation function of the large-scale structure displaying a peak around $150h^{-1}\text{Mpc}$. Here we have used BAO measurements from various astronomical surveys: (i) measurements from the Sloan Digital Sky Survey (SDSS) III DR-12 which report three effective binned redshifts $z = 0.38, 0.51, \text{ and } 0.61$ (Alam et al. 2017), (ii) measurements from the clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample reporting four effective binned redshifts $z = 0.98, 1.23, 1, 52, \text{ and } 1.94$, as in Zhao et al. (2019), (iii) measurements from the high-redshift Lyman- α survey reporting two effective binned redshifts at $z = 2.33$ du Mas des Bourboux et al. (2020) and $z = 2.4$ du Mas des Bourboux et al. (2017). All the measurements are presented in terms of $H(z) \times (r_d r_{d, \text{fid}})$ km $s^{-1}\text{Mpc}^{-1}$, where r_d denotes the co-moving sound horizon and $r_{d, \text{fid}}$ is the fiducial input value provided in the above surveys.

(iv) H0LiCOW sample: Finally, we use the sample from the H_0 Lenses in COSMOGRAIL's Wellspring program,¹ another geometrical probe in this list which measures the Hubble constant in a direct way (without assuming any model in the background). The H0LiCOW collaboration has measured six lens systems, determined by measurements of time-delay distances, $D_{\Delta t}$, between multiple images of strong gravitational lens systems due to elliptical galaxies (Wong et al. 2020). The entire information is encapsulated in the time-delay distance $D_{\Delta t}$. Along with these six systems of strongly lensed quasars, the angular diameter distance to the lens D_l also offers some additional information in terms of four more data points. Therefore, in total, one can employ 10 data points and we have used them in this work (we refer to Birrer et al. 2019; Pandey, Raveri & Jain 2020) for more details in this context).

2.3 Theoretical framework

For a model-independent theoretical description of the dark sectors' interaction, in this work, we follow the similar methodology as in ref. Yang et al. (2015). In the context of a Friedmann–Lemaître–Robertson–Walker universe, we assume that the total energy density of the universe is comprised by DE and DM only where both of them are coupled through a non-gravitational interaction. Thus, the conservation equations for DM and DE are modified as

$$\dot{\rho}_{\text{DM}} + 3H\rho_{\text{DM}} = -Q(t), \quad (1)$$

$$\dot{\rho}_{\text{DE}} + 3H\rho_{\text{DE}}(1+w) = Q(t), \quad (2)$$

where $w = p_{\text{DE}}/\rho_{\text{DE}}$ is the equation of state of DE (p_{DE} denotes the pressure of the DE fluid), $H = \dot{a}/a$ is the expansion rate of the universe and it is related to the total energy density of the universe as $3H^2 = \rho_{\text{DM}} + \rho_{\text{DE}}$ (in the units where $8\pi G = 1$). The function $Q(t)$ describes the interaction between DM and DE, and usually it is taken to be a function of the energy densities of DM and DE. For $Q(t) = 0$ with $w = -1$, the standard ΛCDM cosmology is recovered. Now, combining the conservation equations (1) and (2) with expansion rate of the universe $H(z)$, we obtain Yang et al. (2015):

$$\begin{aligned} -wq &= 2\left(EE'^2 + E^2E'' - \frac{w'}{w}E^2E'\right)(1+z)^2 \\ &\quad - \left[2(5+3w)E^2E' - 3\frac{w'}{w}E^3\right](1+z) \\ &\quad + 9(1+w)E^3, \end{aligned} \quad (3)$$

where for convenience, we have used a dimension-less variable $q = Q(t)/H_0^3$ to characterize the interaction, $E(z) = H(z)/H_0$ is the normalized Hubble rate and the prime denotes the differentiation with respect to the redshift z . Let us note that the symbol q is usually used to represent the deceleration parameter in the literature, but here this symbol has a different meaning as defined above. The detailed derivation of equation (3) is given in Appendix A. Now, using the normalized co-moving distance,

$$D = \frac{H_0}{c} \left(\frac{1}{1+z} \right) d_L(z), \quad (4)$$

where $d_L(z)$ represents the luminosity distance at redshift z , equation (3) can be expressed alternatively as

$$\begin{aligned} -wq &= 2\left(\frac{3D'^2}{D^5} - \frac{D''}{D^4} + \frac{w'D''}{wD^4}\right)(1+z)^2 \\ &\quad + \left[2(5+3w)\frac{D''}{D^4} + \frac{3w'}{wD^3}\right](1+z) \\ &\quad + \frac{9(1+w)}{D^3}. \end{aligned} \quad (5)$$

The above methodology represents a general framework to reconstruct the coupling function with minimal assumption. The only assumption of fact is the validity of the cosmological principle, and a possible coupling between DM and DE has been assumed as a theoretical prior, where this second assumption must be tested with the observational data.

In what follows, we will test this theoretical framework.

3 RESULTS AND DISCUSSIONS

In this section, we present and discuss the results of our analyses considering two separate cases. First, we fix the EoS of DE to $w = -1$, and reconstruct the interaction function $q = Q(t)/H_0^3$. This possibility characterizes a very well-known sub-class of the interaction scenario in the dark sector, known by interacting vacuum energy. Secondly, we consider a very general possibility assuming w as a free and dynamical function, and similarly reconstruct the interaction function q . Thus, having both the possibilities, we explore a very general description of the dark coupling in a model-independent approach. Before we enter into the main results, we rescale the function q (see equation 5) to $\delta(z) = q(1+z)^{-6}$. Such a pre-factor is just considered as a scale transformation with respect to z , introduced to better expose the results in the graphical description. Thus, here onwards the function $\delta(z)$ will characterize the coupling function.

We proceed further considering the above two scenarios of coupling in the dark sector. To reconstruct $\delta(z)$, we use $M_{9/2}$ kernel in all the analyses performed in this work. In the case where we

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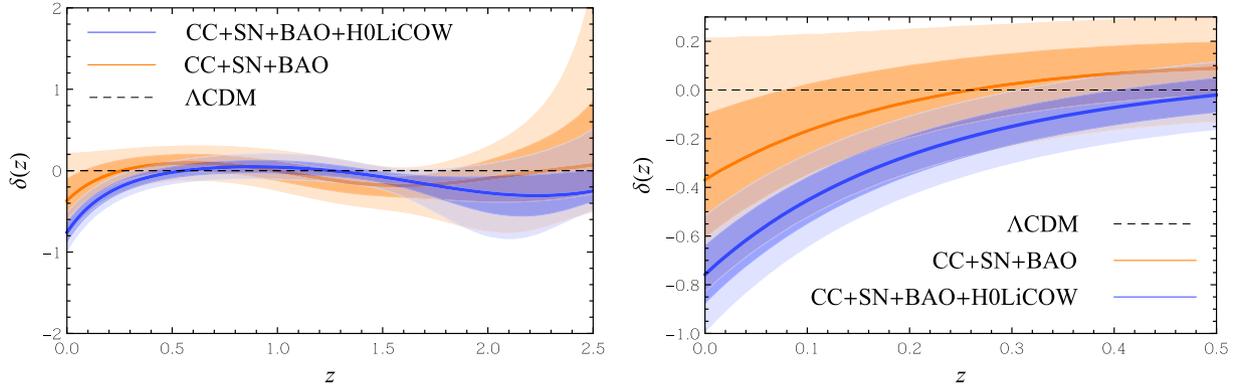


Figure 1. Left-hand panel: Reconstructed coupling function $\delta(z)$ at 1σ and 2σ CL in the interacting vacuum energy scenario from CC+SN+BAO (Orange) and CC+SN+BAO+H0LiCOW (Blue) data. Right-hand panel: The same as in left-hand panel, but restricted to the range $z \in [0, 0.5]$. The dashed black curve corresponds to the canonical Λ CDM prediction and the solid curves represent the GP mean.

assume $w(z)$ to be a free function, we follow the same methodology as presented in Bonilla et al. (2021). For this purpose, we have used modified versions of some numerical routines available in the public GAPP (Gaussian Processes in Python) code (Seikel et al. 2012). In all of our analyses, we employ GP to perform a joint analysis using the minimal data set combination CC+SN+BAO, which to our knowledge has not been investigated previously in the literature. We now present and discuss our main results.

3.1 Interacting vacuum energy

In Fig. 1, we have shown the reconstruction of $\delta(z)$ using the data combinations CC+SN+BAO and CC+SN+BAO+H0LiCOW. In both analyses, we note that for $z > 0.5$ the dynamical coupling function $\delta(z)$ between the dark components is statistically well compatible with $\delta(z) = 0$. It is interesting to note that the GP mean predicts a possible oscillation in $\delta(z)$, where we can note an oscillation between positive and negative values in the analysed range of z . This result strengthens some earlier interaction models having a sign changeable property, see for instance Pan et al. (2019), Pan, Yang & Paliathanasis (2020), Yang et al. (2021). For the present scenario, at late cosmic time, i.e. for $z < 0.5$ ($z < 0.25$), we find a trend towards $\delta < 0$ for CC+SN+BAO+H0LiCOW (CC+SN+BAO) data. When evaluated at present moment, we find $\delta(z=0) = -0.37 \pm 0.24$ (-0.76 ± 0.12) at 1σ CL from CC+SN+BAO (CC+SN+BAO+H0LiCOW) data. This suggests an interaction in the dark sector at more than 3σ CL from the CC+SN+BAO+H0LiCOW joint analysis. It is important to emphasize that these constraints on $\delta(z)$ are subject to the condition $w = -1$. Also, we notice that the combined analysis with several data sets offers a more stringent bound on the interaction function compared to Yang et al. (2015), where only the SN Ia Union 2.1 data set (Suzuki et al. 2012) was employed.

3.2 General interaction scenario in the dark sector

In the previous subsection, we analysed a particular interaction case, namely the interacting vacuum-energy ($w = -1$) to get the constraints on $\delta(z)$. As a second round of analysis, we relax these conditions by assuming $w(z)$ to be a free function. This possibility allows us to reconstruct the coupling in the dark sector in a general way, because in this case, no physical assumption is considered on the EoS of DE. In Fig. 2, we show the reconstruction of $\delta(z)$ from CC+SN+BAO and CC+SN+BAO+H0LiCOW data combinations. Since in this

scenario, we have an additional free parameter w to propagate errors compared to the previous case with $w = -1$, it is expected that large error bars might be imposed on the reconstructed $\delta(z)$ compared to the case with $w = -1$. As a general feature of the GP mean, we can note a flux of energy from DM to the DE at high z , and as cosmic time evolves up to approximately $z < 0.5$, the coupling function $\delta(z)$ reverses its sign, at low z (late time). This again goes in support of some phenomenological models of the interaction (Pan et al. 2019, 2020). In this general framework, we find $\delta(z=0) = -0.31 \pm 0.77$ at 1σ CL from CC+SN+BAO data and $\delta(z=0) = -0.64 \pm 0.43$ at 1σ CL from CC+SN+BAO+H0LiCOW data. These predictions are compatible with the Λ CDM cosmology, i.e. $\delta = 0$.

4 FORECAST FROM GRAVITATIONAL WAVE STANDARD SIRENS

To impose more robust and accurate constraints on the $\delta(z)$ function, we optimize the covariance function using mock gravitational waves (GW) data generated by assuming the Λ CDM model as a fiducial one. As argued in Seikel & Clarkson (2013), for non-parametric regression method such as GP, we aim to generate confidence limits such that the true function is trapped appropriately. But it can be problematic when evaluating functions as w and $\delta(z)$ because they are quantities that depend on the second and third order derivatives of the cosmological observable. We can avoid the problem in identifying an appropriate covariance function which can reproduce expected models accurately by adding simulated data. To accomplish this object, we create a standard sirens mock catalogue, the gravitational wave analogue of the astronomical standard candles, which can provide powerful information about the dynamics of the universe. For a given GW strain signal, $h(t) = A(t)\cos[\Phi(t)]$, the stationary-phase approximation can be used for the orbital phase of inspiraling binary system to obtain its Fourier transform $\tilde{h}(f)$. For a coalescing binary system of masses m_1 and m_2 ,

$$\tilde{h}(f) = \mathcal{Q} \mathcal{A} f^{-7/6} e^{i\Phi(f)}. \quad (6)$$

Here $\mathcal{A} \propto 1/d_L$ is the luminosity distance to the merger's redshift, and $\Phi(f)$ is the binary system's inspiral phase. For more details on the post-Newtonian coefficients and waveforms, one may refer to D'Agostino & Nunes (2019) and Appendix A therein. After defining the GW signal, for a high enough signal-to-noise ratio (SNR), one may obtain upper bounds on the free parameters of the GW signal

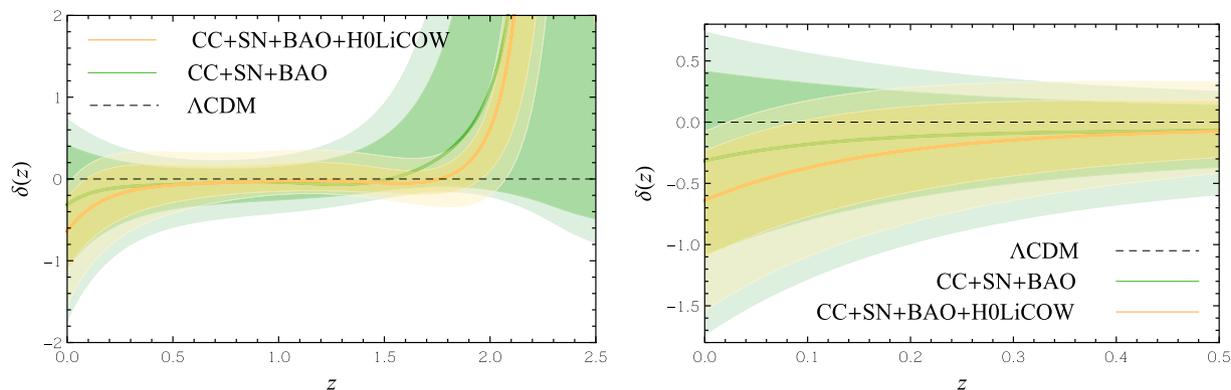


Figure 2. Left-hand panel: Reconstructed coupling function $\delta(z)$ at 1σ and 2σ CL in the general interacting scenario of the dark sectors' from CC+SN+BAO (Green) and CC+SN+BAO+H0LiCOW (Yellow) data. Right-hand panel: The same as in left-hand panel, but restricted to the range $z \in [0, 0.5]$. The dashed black curve corresponds to canonical Λ CDM prediction and the solid curves stand for the GP mean.

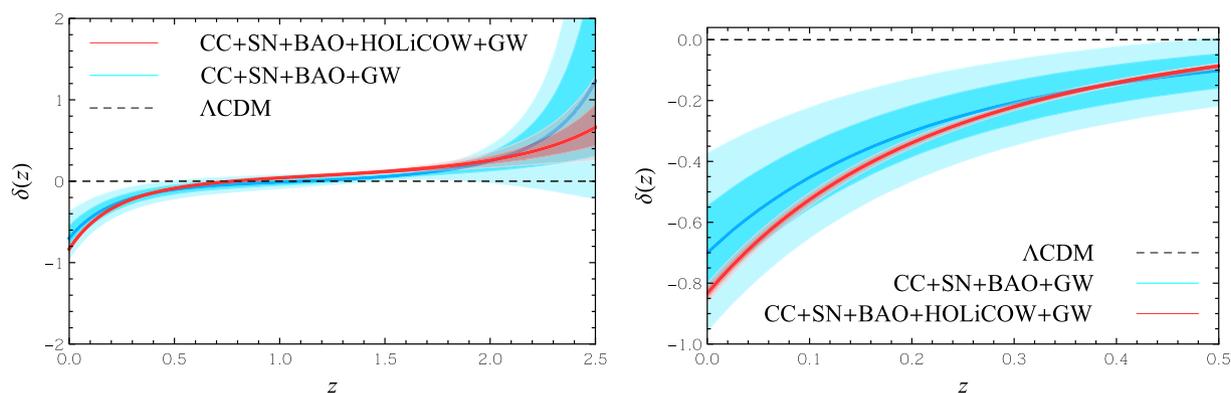


Figure 3. Left-hand panel: Reconstructed coupling function $\delta(z)$ at 1σ and 2σ CL from CC+SN+BAO+GW (Blue) and CC+SN+BAO+H0LiCOW+GW (Red) data, in the interacting vacuum energy scenario. Right-hand panel: The same as in left-hand panel, but restricted to the range $z \in [0, 0.5]$. The dashed black curve corresponds to the canonical Λ CDM prediction and the solid curves are for the GP mean.

$\tilde{h}(f)$ by using the Fisher information analysis. Estimating $d_L(z)$ from GW standard sirens mock data is well established approach, see D'Agostino & Nunes (2019) and references therein. In what follows, we briefly describe our methodology that is used to generate the standard sirens mock catalogue.

In order to generate the mock standard siren catalogue, we consider the ET power spectral density noises. The ET is a third-generation ground detector, and covers frequencies in the range 1–10⁴ Hz. The ET is sensitive to signal amplitude, which is expected to be ten times larger than the current advanced ground-based detectors. The ET conceptual design study predicts BNS detection of an order of 10³–10⁷ per year. However, only a small fraction ($\sim 10^{-3}$) of them is expected to be accompanied by a short γ -ray burst observation. Assuming a detection rate of $\mathcal{O}(10^5)$, the events with short γ -ray bursts will be $\mathcal{O}(10^2)$ per year. In our simulations, 1000 BNS mock GW standard sirens merger events up to $z = 2$ are considered. In the mock catalogue, we have used the input values $H_0 = 67.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_{m0} = 0.31$ for the Hubble constant and matter density parameter, respectively, in agreement with the most recent Planck CMB data (within the Λ CDM paradigm) Planck Collaboration VI (2020). We have estimated the measurement error on the luminosity distance for each event using the Fisher matrix analysis on the waveforms (see ref. D'Agostino & Nunes 2019 for details). We have calculated the SNR of each event, and confirmed that it is a GW detection provided $\text{SNR} > 8$. In what follows, we describe the evolution of the interaction function with the inclusion

of the mock GW standard sirens with the standard cosmological probes.

In Fig. 3, we show the reconstructed interaction function $\delta(z)$ from CC+SN+BAO+GW and CC+SN+BAO+H0LiCOW+GW data combinations for the simple interaction scenario with $w = -1$. When evaluated at the present moment, we find $\delta(z = 0) = -0.70 \pm 0.14$ at 1σ CL for CC+SN+BAO, and $\delta(z = 0) = -0.833 \pm 0.016$ at 1σ CL for CC+SN+BAO+H0LiCOW under the interacting vacuum-energy assumption. Analysing the behaviour of the δ function in the range $z \in [0, 2.5]$, we find an evidence for a sign transition in δ that quantifies the interaction between the dark components. We clearly notice a preference for $\delta < 0$ at late times. In Fig. 4, we show the reconstructed interaction function $\delta(z)$ from CC+SN+BAO+GW and CC+SN+BAO+H0LiCOW+GW data combinations, under the general assumption where $w(z)$ is a free function of z . In this case, we find $\delta(z = 0) = -0.49 \pm 0.69$ at 1σ CL from CC+SN+BAO+GW and $\delta(z = 0) = -0.705 \pm 0.066$ at 1σ CL from CC+SN+BAO+H0LiCOW+GW. In our analysis, even including GW mock data in CC+SN+BAO, we note that δ is compatible with Λ CDM. On the other hand, from CC+SN+BAO+H0LiCOW+GW data, we can notice a prediction for $\delta < 0$ at late times.

It is important to note that the presence of GWs mock catalogue (assuming a fiducial Λ CDM model) is used for the purpose of optimizing the covariance function, as argued previously. The results summarized in Fig. 4 are the most realistic, being the ones for the most general case.

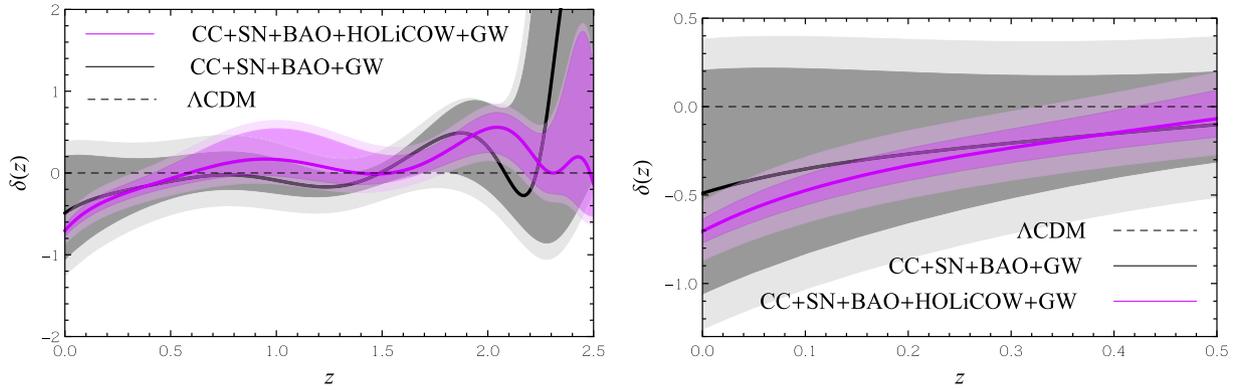


Figure 4. Left-hand panel: Reconstructed coupling function $\delta(z)$ at 1σ and 2σ CL in the general interacting scenario of dark sectors' from CC+SN+BAO+GW (Black) and CC+SN+BAO+HOLiCOW+GW (Purple) data. Right-hand panel: The same as in left-hand panel, but restricted to the range $z \in [0, 0.5]$. The dashed black curve corresponds to the canonical Λ CDM prediction and the solid curves stand for the GP mean.

5 CONCLUSIONS

In this work, we have presented some generalized aspects of dark sectors' interaction that may be of interest to the community: (i) We have investigated the case where DE can assume a dynamical character through its equation of state along with the simplest vacuum-energy case $w = -1$. (ii) We have studied joint analyses of the dark sector interaction with several geometrical probes following a minimally model-dependent way of GP. (iii) We have optimized the covariance function using mock GW standard sirens to better reconstruct the function $\delta(z)$. In short, all these pieces of investigation have led to more general and robust results which could be helpful in order to have a deeper understanding of the physics of the dark sector. Our observations are as follows: We find, for both interacting vacuum and general scenario, that, $\delta(z)$ exhibits a transient nature according to the analyses from CC+SN+BAO and CC+SN+BAO+HOLiCOW data, and at very late time, $\delta(z)$ enters into the negative region (see Figs 1 and 2). This conclusion remains unaltered when we include the 1000 mock GW standard sirens to the above two combined data sets (see Figs 3 and 4). However, the indication of a late interaction is strongly pronounced in the context of the interacting vacuum scenario where we find that for the CC+SN+BAO data, $\delta(z = 0) \neq 0$ at more than 1σ CL, but $\delta(z = 0) \neq 0$ remains valid at more than 3σ CL for CC+SN+BAO+HOLiCOW data. This is an interesting result in this work because the transfer of energy among the dark sector components, which has been observed in some phenomenological models, is not ruled out in light of the model-independent analysis, and also for the combined analyses that we have performed during the reconstruction. However, concerning the general interacting picture, we see that $\delta(z = 0) = 0$ is compatible within 1σ for both CC+SN+BAO and CC+SN+BAO+HOLiCOW data. When the GW standard sirens enter into the analysis, we find that for the interacting vacuum scenario, again $\delta(z = 0) \neq 0$ at several standard deviations, for both CC+SN+BAO+GW and CC+SN+BAO+HOLiCOW+GW data. Whilst for the general scenario, even if for CC+SN+BAO+GW data, $\delta(z = 0) = 0$ is compatible within 1σ but for CC+SN+BAO+HOLiCOW+GW data, we find a strong preference of an interaction at several standard deviations. Summarizing the results, we find that the model-independent analyses indicate for a possible interaction in the dark sector which is strongly preferred for the scenario with $w = -1$. Based on the findings of this study, we believe that it will be worthwhile to investigate, in future communications, various statistical techniques for reconstructing the function $\delta(z)$, such as the

use of neural networks, principal component analysis, and others, which may provide statistical improvements over the standard GP method that we use in the study of cosmological parameters.

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DATA AVAILABILITY

The observational data used in this article will be shared on reasonable request to the corresponding author.

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APPENDIX A: DERIVATION OF THE KEY EQUATION

Here we show the derivation of equation (3) which is the heart of this work. Now, subtracting equation (2) from equation (1) one can obtain

$$[\dot{\rho}_{\text{DM}} - \dot{\rho}_{\text{DE}}] + 3H[\rho_{\text{DM}} - (1 + w)\rho_{\text{DE}}] = -2Q(t). \quad (\text{A1})$$

Now, using the Friedmann's equations $3H^2 = \rho_{\text{DM}} + \rho_{\text{DE}}$ and $2\dot{H} + 3H^2 = -p_{\text{DE}} = -w\rho_{\text{DE}}$, one can express ρ_{DM} and ρ_{DE} , respectively, as

$$\rho_{\text{DM}} = 3H^2 + \frac{2\dot{H}}{w} + \frac{3H^2}{w}, \quad (\text{A2})$$

and

$$\rho_{\text{DE}} = -\frac{2\dot{H}}{w} - \frac{3H^2}{w} \quad (\text{A3})$$

Now differentiating (A2) and (A3) with respect to the cosmic time one can write down:

$$\dot{\rho}_{\text{DM}} = 6H\dot{H} + 2\frac{\ddot{H}}{w} - 2\frac{\dot{H}\dot{w}}{w^2} + 6\frac{H\dot{H}}{w} - 3\frac{H^2\dot{w}}{w^2}, \quad (\text{A4})$$

and

$$\dot{\rho}_{\text{DE}} = -2\frac{\ddot{H}}{w} + 2\frac{\dot{H}\dot{w}}{w^2} - 6\frac{H\dot{H}}{w} + 3\frac{H^2\dot{w}}{w^2}, \quad (\text{A5})$$

Notice that, by changing $\frac{d}{dt} \rightarrow \frac{d}{dz}$, where z denotes the redshift, one can express

$$\dot{w} = -\frac{w'H}{a}, \quad \dot{H} = -(1+z)HH', \quad (\text{A6})$$

where prime denotes the derivative with respect to z . Consequently, one can express \ddot{H} as

$$\ddot{H} = (1+z)^2HH'^2 + (1+z)^2H^2H'' + (1+z)H^2H' \quad (\text{A7})$$

Now, inserting equations (A4), (A5) into equation (A1) and using the derivatives of cosmological parameters with respect to z , and

doing some simple algebraic calculations, one arrives at

$$\begin{aligned} -wQ &= 2\left(HH'^2 + H^2H'' - \frac{w'}{w}H^2H'\right)(1+z)^2 \\ &\quad - \left[2(5+3w)H^2H' - 3\frac{w'}{w}H^3\right](1+z) \\ &\quad + 9(1+w)H^3, \end{aligned} \quad (\text{A8})$$

which if divided by H_0^3 , can be expressed as

$$\begin{aligned} -wq &= 2\left(EE'^2 + E^2E'' - \frac{w'}{w}E^2E'\right)(1+z)^2 \\ &\quad - \left[2(5+3w)E^2E' - 3\frac{w'}{w}E^3\right](1+z) \\ &\quad + 9(1+w)E^3, \end{aligned} \quad (\text{A9})$$

where $q = Q/H_0^3$ and $E = H/H_0$.

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